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ABSTRACT

In the introductory section of this report, the Second International Mathematics Study (1981-82) is described. The study, which involved about 500 classrooms in the United States and classrooms in 24 other countries in grades 8 and 12, was designed to provide detailed information about the content of the mathematics curriculum, how mathematics is taught, and how much mathematics students learn. The intended, implemented, and attained curricula were assessed by questionnaires and achievement tests. Provisions for assuring comparability of samples are presented. Two types of twelfth-grade classes in the United States sample were studied: precalculus classes and calculus classes. The report also includes summaries of data on class size, hours of mathematics instruction, teaching assignments, teacher characteristics, how mathematics teachers spend their time, use of instructional resources, how mathematics students spend their time, homework, use of calculators, what mathematics was taught, untaught topics, and student achievement. Achievement on eight illustrative items is presented, followed by summaries of results on sex differences in achievement, changes in twelfth-grade achievement between the first study (1964) and the second study (1982), and the ranking of the United States internationally. Finally, a summary is included, plus some comments on "A Curriculum Adrift." Figures presenting data are appended. (MNS)

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Student/Class Results from the Second International Mathematics Study from United States Twelfth Grade Classrooms

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Introduction

In 1981-82, students and teachers in about 500 mathematics classrooms across the United States joined their couterparts in some two dozen countries around the world in a comprehensive study of school mathematics. This study was designed to provide detailed information from each country about the content of the mathematics curriculum, how mathematics is taught and how much mathematics students learn. The information is intended to help policy analysts and mathematics educators in individual nations analyze their school programs and identify areas of strength and weakness, and to provide data which are useful to national officials as they plan for future directions in school mathematics in their own countries.

The Study was conducted by members of the International Association for the Evaluation of Educational Achievement, or IEA, an international network of leading educational research institutions.

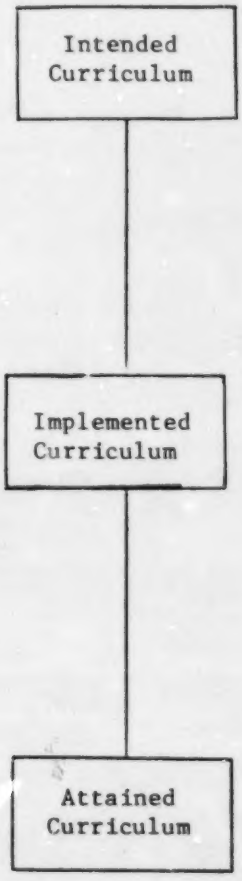
The Study is the second to take place in a twenty-year period. In 1964, twelve countries (identified in Appendix A), including the United States, took part in a similar study. Hereafter, the present study will be called the Second International Mathematics Study, SIMS, of simply "the Study" for short. The 1964 study will be called the First International Mathematics Study of FIMS.

Each country provided its own costs for participating in these studies. In the United States, funding was provided by the National Institute of Education, the National Science Foundation and the National

Center for Education Statistics.

Components of the Second International Mathematics Study

The conceptual design of the Second International Mathematics Study has three components, as represented in the figure below:



The intended curriculum is reflected in curriculum guides, course outlines, syllabi and textbooks adopted at the educational system level. In most countries, nationally defined curricula emanate from a ministry of education or similar national body. In the U.S., such statements of intended goals or specifications of curricular content

are developed in State Departments of Education or at the local school district level. Thus it is considerably more difficult to describe the intended curriculum for the U.A. than for almost any other of the participating countries.

The implemented curriculum focuses on the classroom level where the intended curriculum is taught by the teacher. Teachers may exercise their own judgements in translating curriculum guides or adopted textbooks into a program for their classes. Thus, their selection of topics or patterns of emphasis may not be consistent with those intended. To identify the implemented curriculum, a number of questionnaires were developed for completion by the individual classroom teacher. For example, teachers were asked whether or not they had provided instruction to the target class for each of the items on the achievement tests. They were also asked to provide detailed information on the number of class periods devoted to specific subtopics and which interpretations of selected concepts and processes they utilized. Such highly specific information on curriculum coverage coupled with similarly detailed information on instructional strategies permits a rather comprehensive characterization of what mathematics has been taught and how it was taught to the target populations.

The third component of the study addressed the attained curriculum - what students had learned as measured by the tests and questionnaires. Extensive achievement tests were designed to measure student knowledge and skill in areas of mathematics designated as important and appropriate for each population. The "fit" of these tests to the curriculum in individual coutries varies substantially and is a limiting factor in

international comparisons. Quite obviously, the tests contain items less appropriate in some countries than in others and may not always contain an adequate range of items to fully represent all curricula. This limitation restricts the nature of cross-national comparisons that can be drawn and argues against interpreting the study as an international contest. The student outcome measures also included a number of opinionnaires and attitude scales. These were intended to elicit student views as to the nature, importance, ease, and appeal of mathematics and selected mathematical processes.

This design, as implemented in the U.S., includes the following sources of data:

1. Questionnaires completed by school officials concerning school, teacher and mathematics program characteristics; organizational factors; and school and departmental policies affecting mathematics instruction.
2. Questionnaires completed by teachers to provide background information on experience, training, qualification, beliefs and attitudes. Additional questionnaires to provide general information on instructional patterns (allocated time, ability of class, classroom organization and activities related to individualization of instruction, resources used, and goals and factors affecting instructional decisions), and beliefs about effective teaching. Additional questionnaires related to instruction on selected specific topics.
3. Ratings by teachers (opportunity to learn) of whether the content needed to respond to each item of the achievement tests had been taught that year, in prior years, or not at all, to their students.
4. Questionnaires completed by students providing background information (e.g., parents education and occupation), time spent on homework, and attitudes and beliefs related to mathematics.
5. Achievement tests completed by students at the beginning and at the end of the year.

Target Populations for the Study

The study included a survey of the mathematics curriculum provided for two groups of students, identified as target Population A and B

1. All students enrolled in normal mathematics classes in the U.S. at Year 8 level (Population A).
2. All students enrolled in mathematics classes at Year 12 level in classes requiring as prerequisites two years of algebra and one year of geometry (Population B).

United States Sample

The United States sample in the Second International Mathematics Study was composed of students and teachers from approximately 500 classrooms in about 250 public and private schools across the United States. Students were tested by internationally developed mathematics achievement tests at the beginning of the 1981-82 academic year and again at the end of the school year. The testing also included an attitude questionnaire. These data were collected from approximately 7,500 eighth-grade students and 5,000 students enrolled in twelfth-grade mathematics classes. Technical details of the sample are in Appendix D.

One matter of concern in an international study of schooling is whether the other countries involved have comparable proportions of their students in the target populations defined. At Population A (eighth grade in the U.S.) virtually all students of that age are still involved in schooling in most countries. At Population B

(twelfth grade in the U.S.) there is more variation. Table 1 presents, for several of the countries in the Study, the percentage of the age cohort that is in Population B, the percentage of the grade cohort in Population B and the percentage of the age cohort still in school.

Table 1
Proportion of Population B Students in Relevant Age Groups and Grade for Each Country: 1981

Country	Age Group (Years)	Pop B Percent of Age Group	Pop B Percent of Grade Group	Percent of Age Group in School
Belgium (FL)	17	9-10	25-30	65
British Columbia	17	30	38	82
England & Wales	17	6	35	17
Finland	18	15	38	59
Hungary	17	50	100	50
Israel	17	6	10	60
Japan	17	12	13	92
New Zealand	17	11	67	17
Ontario	18	19	55	33
Scotland	16	18	42	43
Sweden	18	12	50	24
U.S.A.	17	10-12	12-15	85

Notes:
1. Age group is estimated age at middle of school year.
2. While the fourth column represents the percent of the cohort still in school, this does not imply that all these students are in the grade(s) from which the Population B sample is drawn. Thus the second column is not always a simple product of the third and fourth columns.
3. Data are obtained from national reports for each country. The ratio of high school graduates to population age 17 was 72% in the United States in 1981. U.S. data enrollment was based on the school enrollment rates of persons 17 years old according to the October 1981 Current Population Survey.

It can be seen from Table 1 that the U.S. retains in school one of the largest proportions of the age cohorts. In terms of the proportion of students enrolled in college-preparatory mathematics, however, we are retaining a relatively small group -- about 10-12% (see the column in Table 1 headed "Population B percent of age group). Furthermore, a smaller proportion still of the U.S. Population B group is studying calculus (only about 2-3%), than most other countries in the Second International Study.

While there are many interesting findings concerning the Population A classes in the U.S. sample, the present interest of this paper concerns the outcomes observed for the U.S. twelfth grade sample. Suffice it to say in summary that the U.S. Population A students performed at or near the average in the test areas of arithmetic, algebra, and descriptive statistics. In the areas of geometry and measurement, their performance fell at or below the international 25th percentile level.

TWELFTH GRADE FINDINGS

Two Class Types in the Twelfth Grade

To simplify the wide diversity of instructional patterns at the twelfth grade level, the 237 classrooms in the United States sample were classified into precalculus classes and calculus classes. The precalculus classes, 191 in number, were those that focused their efforts on teaching trigonometry, "college" algebra, and analytic geometry as indicated by their responses to specific classroom processes instruments. These classes might best be characterized as "senior mathematics,"

"elementary functions," or one of a group of other titles given to twelfth grade level courses taught to precalculus students. Some of these classes contained as much as a semester of introductory topics in calculus, but fell far short of what would be considered a semester course in the topic at the university level. The calculus classes, 46 in number, were those that were taught a full year course in the calculus, following the outline provided by the College Entrance Examination Board Advanced Placement syllabi for the topic. This was determined by an examination of the textboods indicated and consideration of the classroom process questionnaire for calculus.

The decisions in making these classifications are supported by the different patterns of exposure to content and the different levels of achievement reported later. The data from the teacher questionnaires also show that teachers confirm the differences in the two groups. These, and other data presented later, make a strong case for the use of the two categories for analyzing and describing the curricula, instruction, and achievement in the U.S. twelfth grade classrooms participating in the Study.

Class Size

The student enrollment in the classes studied was composed of 56 percent males and 44 percent females. This same proportion was found in the pre-calculus and the calculus classes having a median size of 19 and the precalculus classes a median size of 21.

Hours of Mathematics Instruction Per Year

The data on time allocated to mathematics instruction were very consistent, with classes in both areas reporting a median period length of 54 minutes. There was a slight difference in the total clock hours of instruction over the year between the two groups, with the precalculus classes receiving a median of 155 clock hours of instruction and the calculus classes 150 clock hours.

Teaching Assignments

The teaching loads at the twelfth grade level varied more than at the eighth grade level. The most typical load was teaching 5 periods per day. However, the precalculus teachers ranged from 2 to 7 periods per day and the calculus teachers from 1 to 6 periods per day. Very few teachers in either category taught anything outside of mathematics. When a subject was taught outside of mathematics, it was most often in some aspect of science.

A large proportion of the calculus teachers also serve as department chairs in mathematics (44 percent compared with 35 percent of precalculus teachers).

Teacher Characteristics

Gender. The teachers participating in the study were almost evenly divided between male (52 percent) and female (48 percent) teachers. However, some differences were noted in their assignments in the two

types of curricula. In the precalculus classes, the ratio of male to female teachers was nearly 1 to 1. In the calculus classes there were almost twice at many male teachers as female (actually, a 7 to 4 ratio).

Age/Experience/Training. The median responses on teachers' backgrounds showed the typical twelfth grade teacher to be 41 years of age, with almost 18 years of teaching experience. Ten of these years had involved the teaching of twelfth grade mathematics. Ten percent of the teachers were between 55 and 65 years of age.

The teachers' responses to questions concerning their collegiate education showed a median of 16 semester courses in mathematics, 2 semester courses in the teaching of mathematics, and 4 semester courses in general methods and pedagogy. About 8 percent of the teachers reported little to no preparation in the areas of the teaching of mathematics and generic methods preparation. However, all of the teachers were reported as being certified in mathematics.

Beliefs About Effective Teaching. The twelfth grade teachers were asked to respond to a set of items dealing with possible components of effective teaching. Here there were differences in the level of endorsement given statements by teachers in the two class types. Seven of the maxims were selected as among the highest in importance by at least 50 percent of the precalculus teachers, while only one such was selected by at least 50 percent of the calculus teachers. The one maxim selected as important by both groups indicated that one should review tests with students shortly after they have been graded. This suggestion was endorsed by 65 percent of

the precalculus teachers and 63 percent of the calculus instructors.

Other statements selected as important by precalculus teachers dealt with making encouraging remarks to students (53 percent), establishing and enforcing clear cut rules of classroom behavior (54 percent), clearing up problems from a previous lesson (60 percent), and avoiding criticizing a student in front of a class (51 percent). These responses were also among the highest rated by calculus teachers, but their response levels indicate a more conservative approach to endorsing general maxims for effective teaching.

Perceptions of the Samples Class. The teachers indicated that their samples classes were perhaps a bit harder to teach than were their normal twelfth grade classes. However, 72 percent of the precalculus and 87 percent of the calculus teachers reported their target classes were either very easy or fairly easy to teach. Both class types reported that about 85 percent of their students were attentive in class and not behavior problems. The remaining students were viewed as being not attentive, but not discipline problems.

When asked to give the number of students in the sampled class which was especially fearful or anxious about mathematics, 78 percent of the precalculus teachers and 82 percent of the calculus teachers reported three or fewer students. Only precalculus classes ever reported more than six students as being especially fearful or anxious about mathematics (10 percent of the precalculus classes).

Goals in Teaching Mathematics. Teachers were asked to rate the relative emphasis given to a set of nine goals characterizing objectives for the teaching of mathematics. Table 2 presents the results of these ratings.

Table 2

Relative Importance of Goals in Teaching Mathematics as Rated by Twelfth Grade Precalculus and Calculus Teachers U.S., 1981-82

(Percents of Teachers)

Goal	Precalculus		Calculus	
	Relatively More Impt.	Relatively Less Impt.	Relatively More Impt.	Relatively Less Impt.
Develop a systematic approach to solving problems	65	5	65	0
Know mathematical facts, principles, and algorithms	42	9	54	7
Understand the logical structure of mathematics	41	11	37	11
Develop an attitude of inquiry	40	6	44	4
Develop an awareness of the importance of mathematics in everyday life	36	19	24	33
Become interested in mathematics	34	14	37	22
Develop an awareness of the importance of mathematics in the basic and applied sciences	32	15	48	9
Perform computations with speed and accuracy	24	23	13	30
Understand the nature of proof	12	38	15	41

The precalculus teachers chose to give relatively more emphasis in their target classes to developing a systematic approach to problem solving; to knowing mathematical facts, principles, and algorithms; and to understanding the logical structure of mathematics.

The calculus teachers indicated they had given relatively more emphasis to developing a systematic approach to problem solving; to knowing mathematical facts, principles, and algorithms; to developing an awareness of the importance of mathematics in the basic and applied sciences; and to developing an attitude of inquiry.

Both of the groups gave their lowest emphasis rating to the goal of understanding the nature of proof. Only 12 percent of the precalculus teachers and 15 percent of the calculus teachers gave it relatively more emphasis than other objectives, while 38 and 41 percent of the teachers, respectively, noted that it received relatively less emphasis than the other goals with their target classes. Note, however, that relative importance of the goals was being sought rather than some sort of absolute importance.

How Mathematics Teachers Spent Their Time

The teachers were asked to estimate the number of minutes per week devoted to generic teaching tasks related to the teaching of their target class. The typical (median) precalculus teacher spent about 2 hours in preparing instruction, 1.5 hours in grading papers, 2 hours in explaining new content to the class, 1 hour in reviewing content

taught previously, and about 25 minutes in dealing with classroom administrative and management details. The median calculus teacher followed much the same pattern, except the time allocations show an additional half-hour of preparation, only 50 minutes for reviewing previously taught material, and 15 minutes on classroom administrative and management tasks. As these responses deal only with a teacher's sampled class, they need to be multiplied by the number of classes taught per day (usually 5 or 6), or at least by the number of "preparations", to get an estimate of the total load placed on a classroom teacher's time.

Overall, the reported use of allocated in-class instructional time consisted of about 40 percent on developing new material, 20 perce on reviewing previously taught material, 10 percent on administrative/ management tasks, and 30 percent on supervising student work in the classroom. (See Figure 1.)

Use of Instructional Resources

In examining instructional resources used, the textbook stood out as the most commonly and consistently used resource. The next most common resource was self-written tests (94 percent), followed by self-writted materials (60 percent) and workbooks and supplementary texts (12 percent). (See Figure 2.)

How Mathematics Students Spent Their Time

Like the eighth grade students, the students in sampled classes at the twelfth grade level spent the major portion of their time listening to teacher presentations (130 minutes per week on the average).

Doing seatwork (60 minutes per week on the average) and taking tests (about 45 minutes per week on the average) accounted for other large blocks of time. (See Figure 3.) While, on average, little time was spent on small group work, there was a great range in time devoted to this use of time across the sampled classrooms. Some classes spent as much as 80 minutes per week on small group activities while others spent virtually none.

Homework

Homework expectations differed markedly between the precalculus and the calculus classes. In precalculus classes, teachers reported that the student in a typical class was expected to do about 4 hours of homework per week, with students in the middle fifty percent of the classes expected to do between 3 and 5 hours of homework per week. In calculus classes, teachers reported that the students were expected to complete 5 hours of homework per week, with the middle fifty percent of classes having from 4 to 8 hours of homework per week.

Use of Calculators

The use of calculators was more prevalent at the senior high school level than at the eighth grade level. About 33 percent of the classes used them 2 or more periods a week. Another 28 percent of the classes used them 1 period a week or less. Teachers of eleven percent of the classes reported that their students did not use calculators, 9 percent indicated that calculators were not allowed in their classes,

and about 20 percent of the teachers failed to provide information.

In the twelfth grade, the most commonly reported instructional uses of calculators in the classroom were checking work, doing homework, and solving mathematics problems in class. In contrast to the eighth graders, who were rarely allowed to use calculators on tests, about 50 percent of the twelfth graders were allowed to use calculators on examinations.

What Mathematics was Taught

Teachers were asked to report whether the content needed to respond to the items of the international test had been taught to their classes. These Opportunity-to-Learn (OTL) data are reported in Table 3.

The results show that the content taught to the precalculus and the calculus groups was different in distinctive ways. The greatest difference between the 191 precalculus classes and 46 calculus classes was, as expected, in elementary functions and calculus, which included differential and integral calculus. In number systems and algebra, for example, the precalculus classes received higher coverage than did the calculus classes. However, as Table 3 also shows, teachers judged that the calculus classes had been taught much of this material in preceeding years. For elementary functions and calculus, relatively little of the material was reported as being taught to the calculus classes prior to twelfth grade, but the content relating to 83 percent of the items was taught during the year.

Table 3, Part 1
Percentage of Cognitive Test Items Taught to Twelfth Grade Students:
U.S., 1981-82
(Average Percent Across Items)

Content Area (N Items)	Precalculus (Number of classes, 191)			Calculus (Number of classes, 46)		
	Taught Before	Taught This Year	Never Taught	Taught Before	Taught This Year	Never Taught
Sets & Relations (7items)	31	50	19	50	40	10
Number Systems (17 items)	39	42	19	75	14	11
Algebra (26 items)	34	52	14	53	41	6
Geometry (26 items)	21	40	39	41	26	33
Elementary Functions & Calculus (46 items)	8	37	55	9	83	8
Probability and Statistics (7 items)	29	14	57	50	6	44
Finite Mathematics (4 items)	29	21	50	62	8	30

(Table 3 continued on next page)

Table 3, Part 2
Percentage of Cognitive Test Items Taught to Twelfth Grade Students:
U.S., 1981-82
(Average Percent Across Items)

Content Area (N Items)	All Classes (Number of classes, 237) Taught Before	Taught This Year	Never Taught
Sets & Relations (7 items)	35	48	17
Number Systems (17 items)	46	37	17
Algebra (26 items)	38	50	12
Geometry (26 items)	25	37	38
Elementary Functions & Calculus (46 items)	8	46	46
Probability and Statistics (7 items)	33	12	55
Finite Mathematics (4 items)	35	18	47

Mathematics Not in the Twelfth Grade Curricula

The development of the cognitive tests for the Second Study strove to provide items which were of current curricular interest to the countries participating. This process worked from a logical grid of curricular areas and topics common to most of the countries taking part. However, it was impossible to write an examination that fitted each of a large number of different curricular programs perfectly. As a result there were items which some classes of students in the U.S. and in each other country had little or no opportunity to study.

In the present case, this method affected twelfth grade classes in the United States more often in the areas of geometry, probability and

statistics, and finite mathematics than in the other areas shown in Table 3. In addition, the lack of opportunity to learn also affected the students in the precalculus classes in the elementary functions and calculus subtest. An overall comparison measure of this opportunity to learn showed the United States mean percentage of items actually taught (twelfth grade or before) to be about 65 percent overall and the international mean percentage for items actually taught to be 73 percent.

The test content items falling outside the United States curriculum generally involved topics in linear algebra (transformations and matrix algebra), parametric equations, combinatorics, and, for the precalculus classes, advanced topics in differentiation and integration.

Student Achievement in Twelfth Grade Mathematics

Table 4 contains achievement results for the twelfth grade by class types.

Table 4, Part 1
Twelfth Grade Achievement by Subtopic: U.S. and International, 1981-82
(Mean Percent Correct)

Content Area	United States								International* Posttest
	Precalculus (Number of classes, 191)				Calculus (Number of classes, 46)				
	OTL	Pre	Post	Difference	OTL	Pre	Post	Difference	
Sets & Relations	81	48	54	+6	90	66	64	-2	62
Number Systems	81	33	38	+5	89	43	48	+5	50
Algebra	87	35	40	+5	94	53	57	+4	57
Geometry	60	24	30	+6	67	35	38	+3	42
Elem. Functions/ Calculus	45	18	25	+7	92	26	49	+23	44
Probability & Statistics	43	36	39	+3	57	48	48	0	50
Finite Mathematics	50	24	29	+5	71	36	38	+2	44

* The countries included, in addition to the U.S., are: Belgium (Flemish); Belgium (French); Canada (British Columbia; Canada (Ontario); England and Wales; Finland; Hong Kong; Hungary; Israel; Japan; New Zealand; Scotland; Sweden; Thailand.

Table 4 continued on next page.

Table 4, Part 2
Twelfth Grade Achievement by Subtopic: U.S. and International, 1981-82
(Mean Percent Correct)

Content Area	United States All Classes (Number of classes, 237)				International Posttest
	OTL	Pre	Post	Difference	
Sets & Relations	83	52	56	+4	62
Number Systems	83	35	40	+5	50
Algebra	88	38	43	+5	57
Geometry	62	26	31	+5	42
Elementary Functions/ Calculus	54	19	29	+10	44
Probability & Statistics	45	39	40	+1	50
Finite Mathematics	53	27	31	+4	44

Figures 4 and 5 portray in detail the results for two topic areas, algebra and analysis (elementary functions and calculus). Figure 6 makes the international comparisons somewhat more obvious.

The data in Table 4 show the differences that existed between the students in the precalculus and calculus classes. Even in the instances where the calculus classes had little exposure to the content, their performance was still higher.

We note furthermore that in every content area (sets and relations, number systems, algebra, etc.) the end of the year average achievement of the precalculus classes was less (and in many cases considerably less than the beginning of the year achievement of the calculus students.

It is important to observe that the great majority of U.S. senior high school students in fourth and fifth year mathematics classes (that is, those in precalculus classes) had an average performance level that was at or below that of the lower 25 percent of the other countries. The end-of-year performance of the students in the calculus classes was at or near the international means for the various content areas, with the exception of geometry. Here, U.S. performance was below the international average. That is to say, the achievement of the highest scoring U.S. mathematics classes (2-3 percent of twelfth grade students) was at best comparable to the performance of the overall Population B classes in other countries. This was in spite of the fact that the U.S. calculus students represent a smaller fraction of the age cohort than is represented in the Population B of most of the other countries in the Second Study.

Achievement by Item

Here are eight illustrative items from the twelfth grade tests. Item 001 deals with operations on sets. Notice that there was considerable improvement from 1964 to 1982 on this item (perhaps reflecting its far more widespread inclusion in the mathematics curricula of 1982). Item 002 involves some knowledge of number system properties used to simplify what is essentially an algebraic expression. Performance is somewhat low.

Item 006 dealing with zeroes of a function, showed considerable gain (especiallythe precalculus classes) during the year. On item 054,

on analytic geometry, performance was quite low, far below the international median, and roughly comparable to U.S. performance on the same item in 1964.

Item 039, dealing with trigonometry, had a beginning of the year score less than that expected by guessing. Indeed, an examination of the wrong responses showed that about 1/3 of the students chose distractor E, indicating the assumption that the function is multiplicative. End of year performance on this item was also disappointingly low.

Item 030 is a probability item for which performance was somewhat below the international average but slightly above that of 1964. Item 029, a typical calculus item dealing with the concept of an integral, proved difficult for all students, even for those in the full year calculus courses. Item 044 is considerably harder that 029, reflecting little understanding of the concept of integration.

Item 001

If X and Y are sets, then $(X \cup Y) \cap (X \cap Y)$ is equal to

A X

B Y

C $X \cup Y$

D $X \cap Y$

E $(X \cup Y) \cup (X \cap Y)$

	OTL		Percent Correct	
	During 12th Grade	Through 12th Grade	Beginning	End
Precalculus	25	91	54	52
Calculus	2	95	73	70
Overall			57	56
Overall Score in 1965				21
International Median				64

Item 002

1. For all rational numbers a, b, c, and d, $a - (b + (c - d))$ is equal to

A $a - b + c - d$

B $a - b - c + d$

C $a - b - c - d$

D $a - b + c + d$

E None of these

	OTL		Percent Correct	
	During 12th Grade	Through 12th Grade	Beginning	End
Precalculus	36	100	50	58
Calculus	29	100	62	73
Overall			52	61
International Median				81

Item 006

The curve defined by $y = 3x(x - 2)(2x + 1)$ intersects the x-axis only at the points

A $(-2,0)$ and $(\frac{1}{2},0)$

B $(2,0)$ and $(-\frac{1}{2},0)$

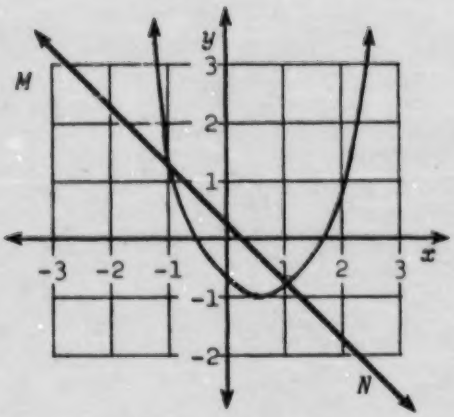
C $(3,0)$ and $(-2,0)$ and $(\frac{1}{2},0)$

D $(3,0)$ and $(2,0)$ and $(-\frac{1}{2},0)$

E $(0,0)$ and $(2,0)$ and $(-\frac{1}{2},0)$

	OTL		Percent Correct	
	During 12th Grade	Through 12th Grade	Beginning	End
Precalculus	80	98	26	44
Calculus	76	100	53	72
Overall			31	49
International Median				73

Item 054



For what values of x does the function represented by the straight line $\overleftrightarrow{MN}$ exceed the quadratic function?

A $-1 < x < 1$

B $x < -1$ or $x > 1$

C $-\frac{3}{4} < x < 1\frac{1}{4}$

D $x > 0$

E $x > y$

	OTL		Percent Correct	
	During 12th Grade	Through 12th Grade	Beginning	End
Precalculus	62	86	22	31
Calculus	62	98	30	44
Overall			24	33
Overall Score in 1964				29
International Median				58

Item 039

If $\cos \theta = \frac{1}{2}$, then $\cos 2\theta$ is equal to

A $-\frac{1}{2}$

B $\frac{1}{2}$

C $-\frac{\sqrt{3}}{2}$

D $\frac{\sqrt{3}}{2}$

E 1

	OTL		Percent Correct	
	During 12th Grade	Through 12th Grade	Beginning	End
Precalculus	83	96	11	32
Calculus	63	100	38	44
Overall			16	34
International Median				64

Item 030

A set of 24 cards is numbered with the positive integers from 1 to 24. If the cards are shuffled and if only one is selected at random, what is the probability that the number on the card is divisible by 4 or 6?

A $\frac{1}{6}$

B $\frac{5}{24}$

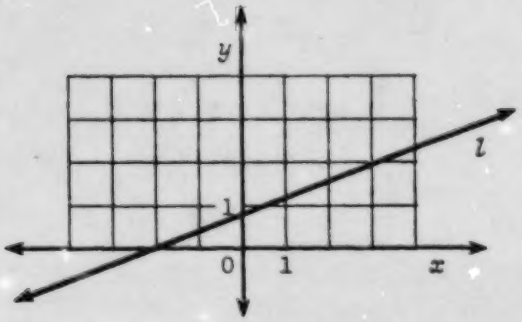
C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{5}{12}$

	OTL		Percent Correct	
	During 12th Grade	Through 12th Grade	Beginning	End
Precalculus	22	38	46	48
Calculus	9	61	63	56
Overall			50	49
Overall Score in 1964				45
International Median				58

Item 029



The line l in the figure is the graph of $y = f(x)$.

$\int_{-2}^{3} f(x)\,dx$ is equal to

A 3

B 4

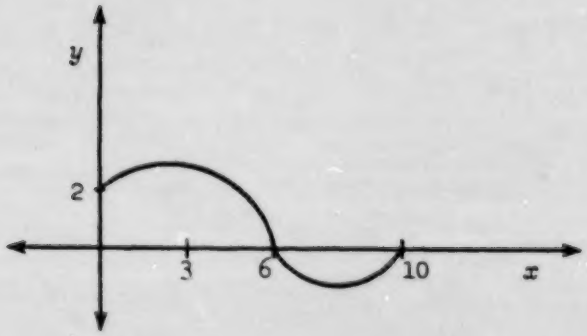
C 4.5

D 5

E 5.5

	OTL		Percent Correct	
	During 12th Grade	Through 12th Grade	Beginning	End
Precalculus	16	16	18	26
Calculus	100	100	22	53
Overall			19	31
International Median				59

Item 044



The graph of the function f is shown above for $0 \leq x \leq 10$.

$\int_0^a f(x)\,dx$ attains its greatest value when a is equal to

A 0

B 2

C 3

D 6

E 10

	OTL		Percent Correct	
	During 12th Grade	Through 12th Grade	Beginning	End
Precalculus	17	18	10	12
Calculus	96	96	7	26
Overall			9	15
International Median				28

Sex Differences in Achievement in the Twelfth Grade

The data in Table 5 show the performance of the male and female students taking part in SIMS at the twelfth grade level. The data used in this analysis were the unweighted student scores by item for each sex. Unlike the findings for the eighth grade level, the data here illustrate a pattern of higher performance on the cognitive items for the male students involved in the study. In particular, the results show that the posttest mean for each subtest was higher for the male students than for the female students. Male change scores usually equaled or exceeded (often only marginally) the female change scores on each of the cognitive subtests considered. The size and consistency of these differences may be taken as suggesting one or more sex-related variables at work.

Table 5
Twelfth Grade Mathematics Achievement by Gender: U.S., 1981-82
(Unweighted Student-Level Data)

	(Number of Students, Female) 1999			(Number of Students, Male) 2559		
Topic	Pre	Post	Difference	Pre	Post	Difference
Sets/Relations	53	55	+2	50	57	+7
Number Systems	33	38	+5	35	41	+6
Algebra	37	42	+5	39	45	+6
Geometry	24	29	+5	27	32	+5
Elementary Functions and Calculus	18	28	+10	20	31	+11
Probability and Statistics	35	35	0	41	44	+3

Changes in Twelfth Grade Achievement Between 1964 and 1982

Twenty of the 136 twelfth grade cognitive test items were included in both the First IEA Mathematics Study and Second IEA Mathematics Study with no significant changes. Another six items could potentially be selected for comparisons but were substantially altered and were not chosen for comparisons here. Table 6 presents the results for twenty items clustered by the subtest topics for twelfth grade. The number of items in each cluster is given in parentheses after the name of the cluster.

Table 6
Twelfth Grade Posttest Achievement by Topic on Common Items for 1964 and 1982: U.S.

Topics (Number of Items)	1964	1982	Change
Sets and Relations (1)	21	56	+35
Number Systems (1)	45	59	+14
Algebra (3)	44	45	+ 1
Geometry (4)	36	37	+ 1
Elementary Functions/ Calculus (10)	25	31	+ 6
Probability and Statistics (1)	45	49	+ 4
Overall (20)	32	38	+ 6

These data are portrayed in Figure 7.

The data show a slight overall increase in student performance from 1964 to 1982. The only topic in which significant change was found was that of elementary functions and calculus. Here there was a gain of six percent on a ten item subtest. It is possible, however,

that this gain reflects the fact that greater proportions of students took calculus in 1982 than in 1964. (The number of students taking the Advanced Placement Calculus examination has been reported by the College Entrance Examination Board as going from 8,000 in 1964 to 32,000 in 1982.) There is also the possibility that the 1982 Population was more selective than was the corresponding Population in 1964. At any rate, the comparative data suggest that for the subject matter represented by the items in Table 6, the twelfth grade college preparatory mathematics students in the U.S. have not lost ground in the eighteen years between the First and Second International Mathematics Studies.

In order to provide an indication of how much of the gain was provided by the 46 calculus classes in the twelfth grade sample, Table 22 presents the Second Study posttest results separated for those in calculus classes and those in precalculus classes. The First Study made no such distinction so only one set of scores can be reported from it and these scores serve as the common basis for computing change across the two types of SIMS classes.

Table 7
Twelfth Grade Posttest Achievement for Common Items by Class Type: U.S., 1964 and 1982

		Precalculus (Number of Classes, 191)		Calculus (Number of Classes, 46)	
Topics (Number of Items)	1964	1982	Change	1982	Change
Sets & Relations (1)	21	52	+31	70	+49
Number Systems (1)	45	56	+11	72	+27
Algebra (3)	44	43	- 1	59	+15
Geometry (4)	36	36	0	45	+ 9
Elementary Functions/ Calculus (10)	25	27	+ 2	50	+25
Probability and Statistics (1)	45	48	+ 3	56	+11
Overall (20)	32	35	+ 3	53	+21

These data suggest that much of the overall growth in Table 6 can be attribued to the 46 Second Study calculus classes. However, even the precalculus students showed a slight increase overall. The gain of 2 percentage points in elementary functions and calculus may indicate that even our less prepared precalculus students have made some advances in that topic area. The lack of growth of the precalculus students in algebra and geometry is cause for concern.

As in the eighth grade, it is of interest whether the changes took place primarily in less cognitively demanding tasks of computation or in relatively more cognitively demanding tasks of comprehension, analysis and application. Table 8 presents the data from the twenty anchor items clustered in the four cognitive levels described earlier.

Table 8
Twelfth Grade Posttest Achievement by Cognitive Level on Common Items for 1964 and 1982: U.S.

Level (Number of Items)	1964	1982	Change
Computation (6)	45	50	+ 5
Comprehension (7)	26	38	+12
Application and Analysis (7)	26	27	+ 1
Overall (20)	32	38	+ 6

Figure 8 portrays the increase or decrease of each common item with the items arranged by cognitive level.

The gain of 12 percentage points in comprehension appears to be noteworthy given that there were seven items in that cluster. However, six of the seven items are from the area of elementary functions and calculus which has already been seen to elicit a significantly higher level of performance by the calculus classes than the precalculus classes. In order to get some feel for how much of the gain is due to the calculus classes, Table 9 breaks the three cognitive-level cluster results down by class type.

Table 9
Twelfth Grade Achievement on Common Items by Cognitive Level and Class Type: U.S., 1964 and 1982

		Precalculus (Number of Classes, 191)		Calculus (Number of Classes, 46)	
Level (Number of Items)	1964	1982	Change	1982	Change
Computation (6)	45	48	+ 3	59	+14
Comprehension (7)	26	33	+ 7	61	+35
Application and Analysis (7)	26	25	- 1	39	+13
Overall (20)	32	35	+ 3	53	+21

The data in Table 9 show a rather stable performance for pre-calculus students in computation and application and a sizeable gain in comprehension. The pattern of gain is more dramatic and consistent in all areas for the calculus students but it is somewhat misleading to compare 1982 calculus students with a general group of students enrolled in the fourth year of secondary school mathematics in 1964 which included some calculus classes but also many others.

In spite of all necessary qualifying remarks, the pattern that emerges from Tables 6 through 9 is one of stability and mild gains, especially in analysis and in comprehension. Our best have held their own or become somethat better in the last twenty years.

Where Does the U.S. Stand Internationally? (Twelfth Grade)

Figures 9 to 14 show the rank ordering of the countries taking part in the Population B study for the topics of sets and relations,

number systems, algebra, geometry, probability and statistics, and elementary functions and calculus. The following observations can be made about the "staircase" graphs:

1. In some topics the U.S. calculus classes are among the upper half of the countries and are always higher than the U.S. precalculus classes.

2. In most topics the U.S. precalculus classes are among the bottom four or five countries.

3. The overall results, combining the two class types, do not differ markedly from the results for precalculus classes.

Table 10 presents the mean posttest data for the U.S. overall, for U.S. precalculus and calculus classes separately and the means for the international posttest.

Table 10
Twelfth Grade Posttest Achievement Means: U.S. and International, 1981-82
(Mean Percent Correct)

Topic	United States			Fifteen Countries*
	Precalc.	Calculus	Total	Posttest Mean
Sets and Relations	54	64	56	62
Number Systems	38	48	40	50
Algebra	40	57	43	57
Geometry	30	38	31	42
Elementary Functions/ Calculus	25	49	29	44
Probability and Statistics	39	48	40	50
Finite Mathematics	29	38	31	44**

* The countries included, in addition to the U.S., are: Belgium (Flemish); Belgium (French); Canada (British Columbia); Canada (Ontario); England and Wales; Finland; Hong Kong; Hungary; Israel; Japan; New Zealand; Scotland; Sweden; Thailand.

** Estimated median score

Table 10 indicates that the total U.S. sample performs at a level markedly lower than the mean level of mathematics in countries participating in SIMS. The calculus classes in the U.S. sample performed at or near the mean level of the same comparison group internationally.

Table 11 presents a comparison of the U.S. posttest mean with international quartile and median data for all subtests except finite mathematics (for which the data were not available).

Table 11
Mean Achievement for U.S. Classes, Medians and Quartiles
for 14 Countries: Twelfth Grade, 1981-82
(Percent Correct)

Topic	U.S. Means			International		
	Total	Precalc.	Calculus	25th Percentile	Median	75th Percentile
Sets & Relations	56	54	64	51	61	72
Number Systems	40	38	48	40	47	59
Algebra	43	40	57	47	57	66
Geometry	31	30	38	33	42	49
Elementary Functions/Calculus	29	25	49	28	46	55
Probability/ Statistics	40	39	48	38	46	64

The U.S. precalculus classes in no topic reached the international median and in most cases were closer to the 25th percentile. The U.S. calculus classes were in most areas above the median but in no topic were they at the 75th percentile.

Summary

An international study of educational achievement provides the unique opportunity to "step" outside and get a braoder picture of teaching and learning in one's own country. The Second International Mathematics Study provides such a perspective.

For twelfth grade college preparatory mathematics, two main programs were found. Precalculus classes studied typical senior high school topics such as trigonometry, college algebra, and analytic geometry. Calculus classes followed the Advanced Placement Calculus syllabus. The calculus classes, containing many of our nation's best secondary students, performed at a level (especially in elementary functions and calculus) which compares favorably with international averages. However, the students in the standard twelfth grade college preparatory classes (called "precalculus") achieved at levels which were exceeded, on average, by about 75 percent of the countries in the Second International Study who tested at the Population B level.

The twelfth grade program was built upon a foundation that is by most international standards highly compartmentalized. In the U.S., high school mathematics typically consists of one year of algebra, one year of geometry, another year of algebra, and then more advanced topics such as analytic geometry, trigonometry, and concepts of calculus in the fourth year. In most countries of the world, a more integrated approach to mathematics is taken, one in which the subject is presented in a more cohesive manner with unification of the topics according to major relationship, not arbitrary placement according to a course name classification. It is plausible that the "fragmentation" and "low intensity" found in many of our mathematics programs

could be allayed by a more integrated approach to the high school mathematics curriculum.

This portrait of twelfth grade mathematics in the U.S. is also one of diversity in time allocations and subtopic coverage. The students in the Advanced Placement Calculus classes are achieving at about the international average. However, these students represent only a small proportion (about 2-3) percent) of seventeen year olds. This select group of students, the "cream of the crop", could reasonably be expected to do much better on the items on the international test than they did, especially those items that are contained in the elementary functions and calculus subtest.

The precalculus classes provide another view. This group, the majority of our twelfth grade college-preparatory mathematics students, looks quite weak internationally, often scoring among the lowest one-fourth of the countries participating. As we have seen, these students, on average, know less mathematics at the end of the twelfth grade than do the students in the majority of our calculus classes at the beginning of their twelfth grade years. Clearly, the majority of our twelfth grade students are not achieving at a level that will enable them to do well in college-level mathematics courses.

A Curriculum Adrift?

Many reasons may be offered for the present state of twelfth grade mathematics. There is, for example, a lack of cohesion in senior high school mathematics. It is difficult to describe, in precise terms, what twelfth grade mathematics entails. While on one hand, it could be said that this reflects efforts to provide, at the local level, for individual needs and differences, it may be more indicative of a curriculum adrift without direction or purpose.

It is the opinion of some that the widespread use of generalized measures of mathematical aptitude for college entrance, rather than curricular specific mathematics tests, is exacerbating this problem of a weakly focused senior high school mathematics curriculum. Students in the college-preparatory program who are interested in scoring well on these widely used college aptitude examinations do not study their high school syllabi for trigonometry, or analytic geometry, or the like. Rather, they go to manuals devised to prepare them for the more generalized skills needed for the standardized tests. Further more, they take these tests in their junior year. This early testing may result in many students viewing the twelfth grade as more of a "holding bay" than a time for the sustained, serious study of mathematics.

An additional explanation for the effect of testing on the curriculum resides in the fact that only one curriculum specific test for twelfth grade mathematics exists--the AP Calculus examination. The previous data show that where a strong "national" core curriculum exists and students are aware of the expectations, higher levels of achievement result. Teachers know the core topics that are valued and the expectations for student achievement on each. In the precalculus mathematics sequence, we have no such beacon to provide leadership for our teachers or schools.

This question of a "national" curriculum also surfaces in the comparison of time usage, intensity of topic coverage, and other phases of mathematics instruction which hold potential for influencing student achievement. At present, we are trying to meet the needs of students heading to careers in science related fields, who need calculus; students heading to careers in business and the social sciences, who need introductory probability and statistics and some elementary linear algebra; and

students who are heading to careers in technology and computer science, who need work in discrete mathematics. The present precalculus curriculum seems to be too heavily oriented toward the first group to be of great advantage to students in the latter two groups. Further, attempts to modify the program to meet the needs of the other students would only lead to the weakening of the calculus preparatory sequence. Thus, at least two separate courses need to be developed for this level. We have reached a point where two mathematics sequences may be needed at the twelfth grade level. Further, we must consider starting the algebra sequence in the eighth grade on a uniform basis for all students capable of studying four years of today's mathematics secondary curriculum.

The implications which may be drawn from this material, and other findings, resulting from the Study are far-reaching. The major points deal with curriculum and time usage in that curriculum. The expectations both of teachers and students need to be increased. For the teachers, this change will call for a greater emphasis on the development of topics and the tying of them together using major unifying themes. This will call for renewed efforts to develop comprehensive and focused curricular materials. For students, the changes suggested will require more study of mathematics, more coverage, and more work.

At a national level, the changes needed include a common consensus on the contents of the mathematics curriculum, the development of "national" tracks within that curriculum which lead to both precalculus and discrete-probability related programs at the collegiate level. The opportunity to respond to these needs provides mathematics educators and those concerned with the status of school mathematics an opportunity which can only be ignored only at considerable peril from a national and human resources viewpoint.

Appendix A

List of Participating Countries

Austrialia*	Israel*
Belgium (Flemish)	Ivory Coast
Belgium (French)*	Japan*
Canada (British Columbia)	Luxembourg
Canada (Ontario)	Netherlands*
Chile	New Zealand
Dominican Republic	Nigeria
England and Wales	Scotland*
Finland*	Swaziland
France*	Sweden*
Hong Kong	Thailand
Hungary	United States*
Ireland	

*These countries also participated in the First International Mathematics Study in 1964. The Federal Republic of Germany took part in the First Study only.

Figures

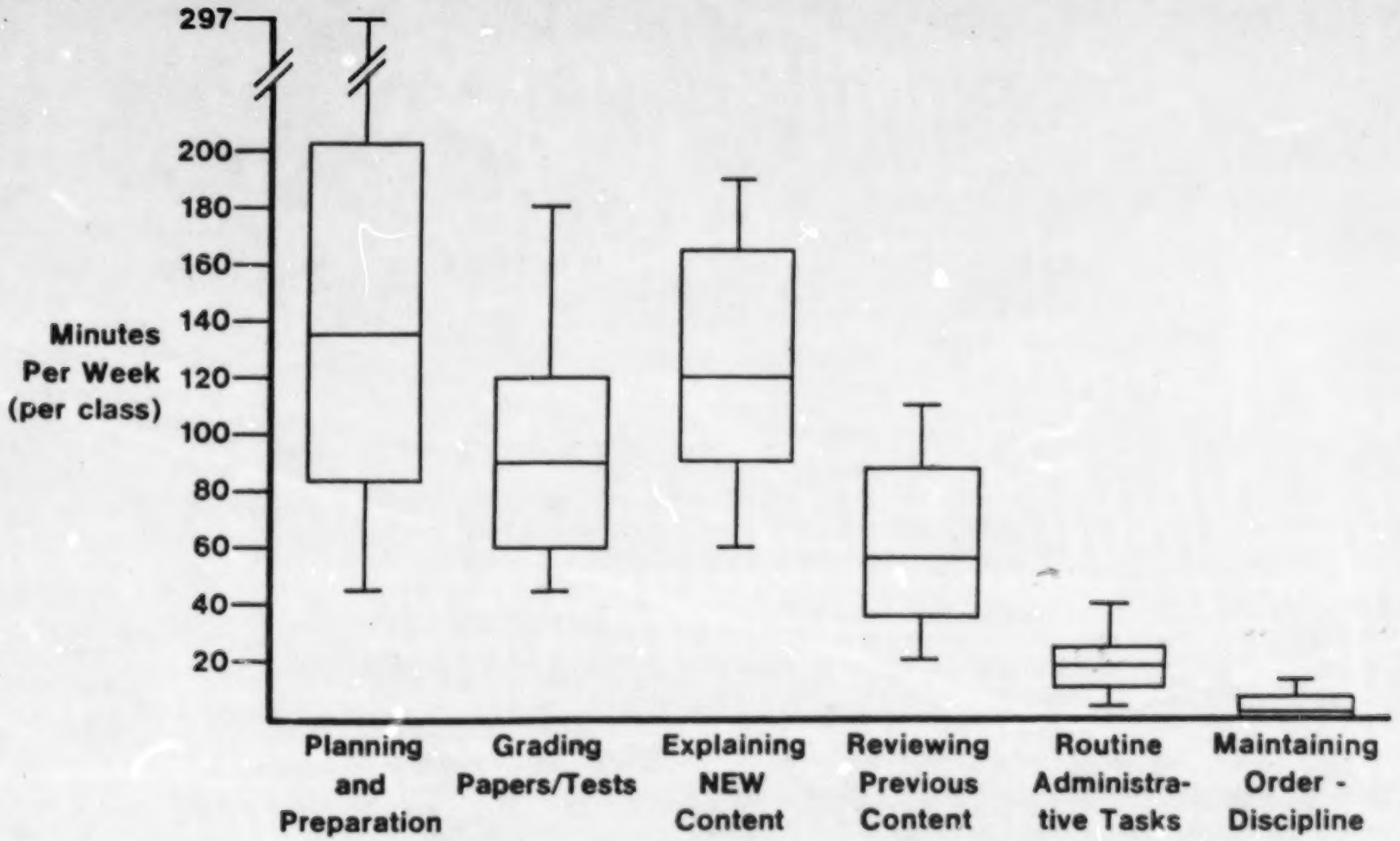


Figure 1 How Teachers' Time is Spent on Selected Teaching Activities in Twelfth Grade Mathematics.

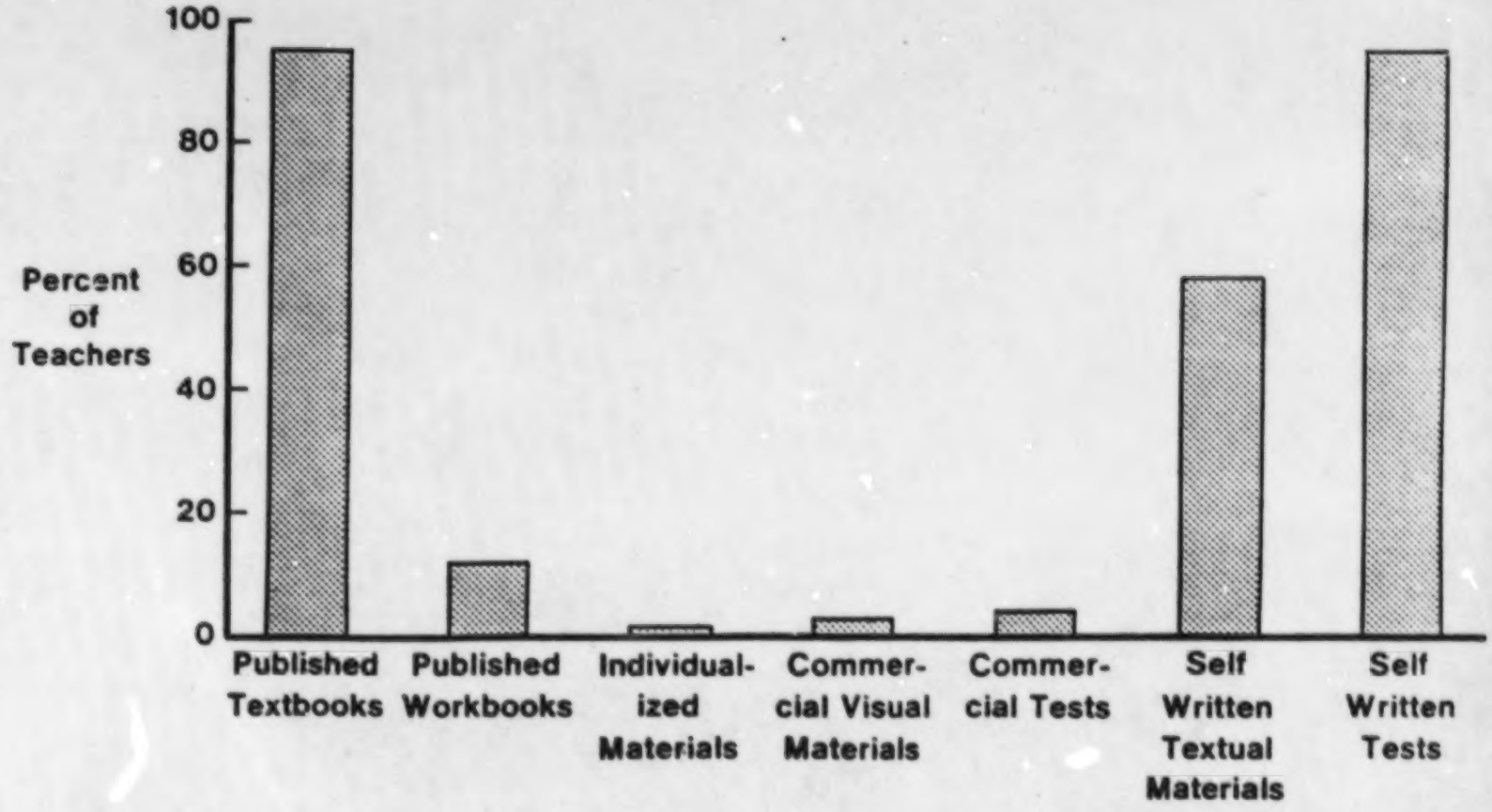


Figure 2 Percent of Teachers Using Various Instructional Resources in Twelfth Grade Mathematics Classes

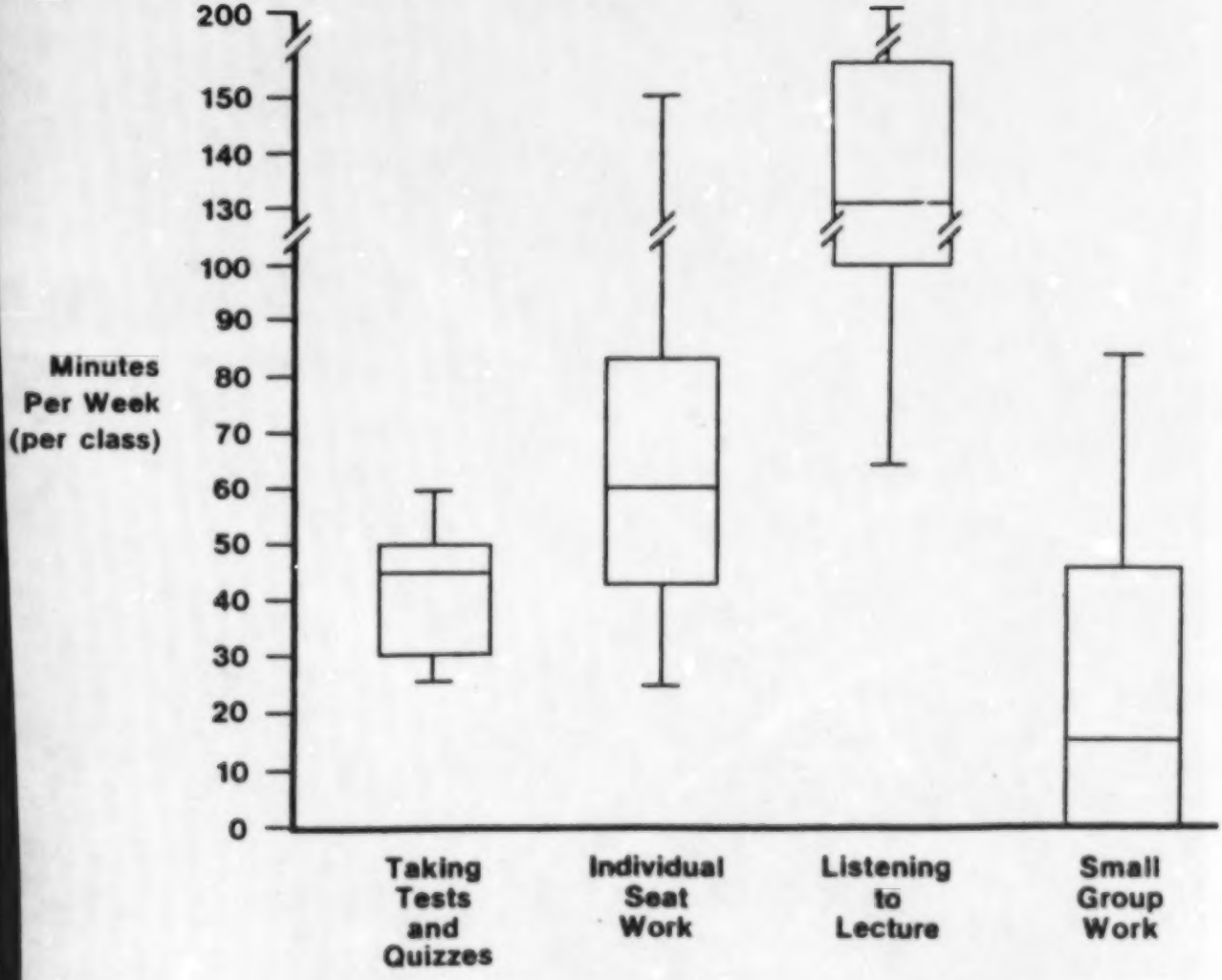


Figure 3 Amount of Student Time Spent in Selected Activities in Twelfth Grade Mathematics Classes.

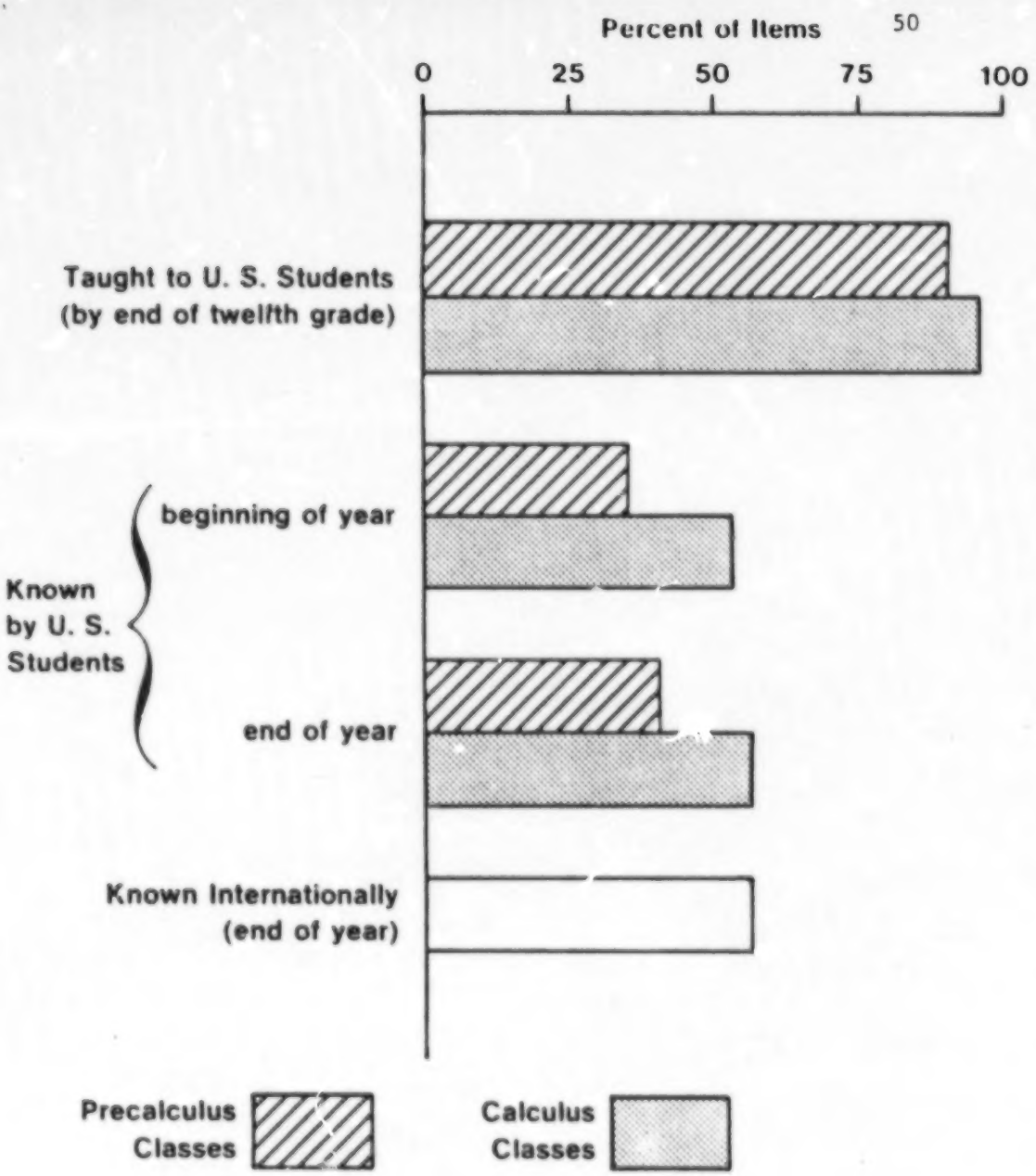


Figure 4 Average Percent of ALGEBRA Items on International Test Taught and Learned at End of Secondary School

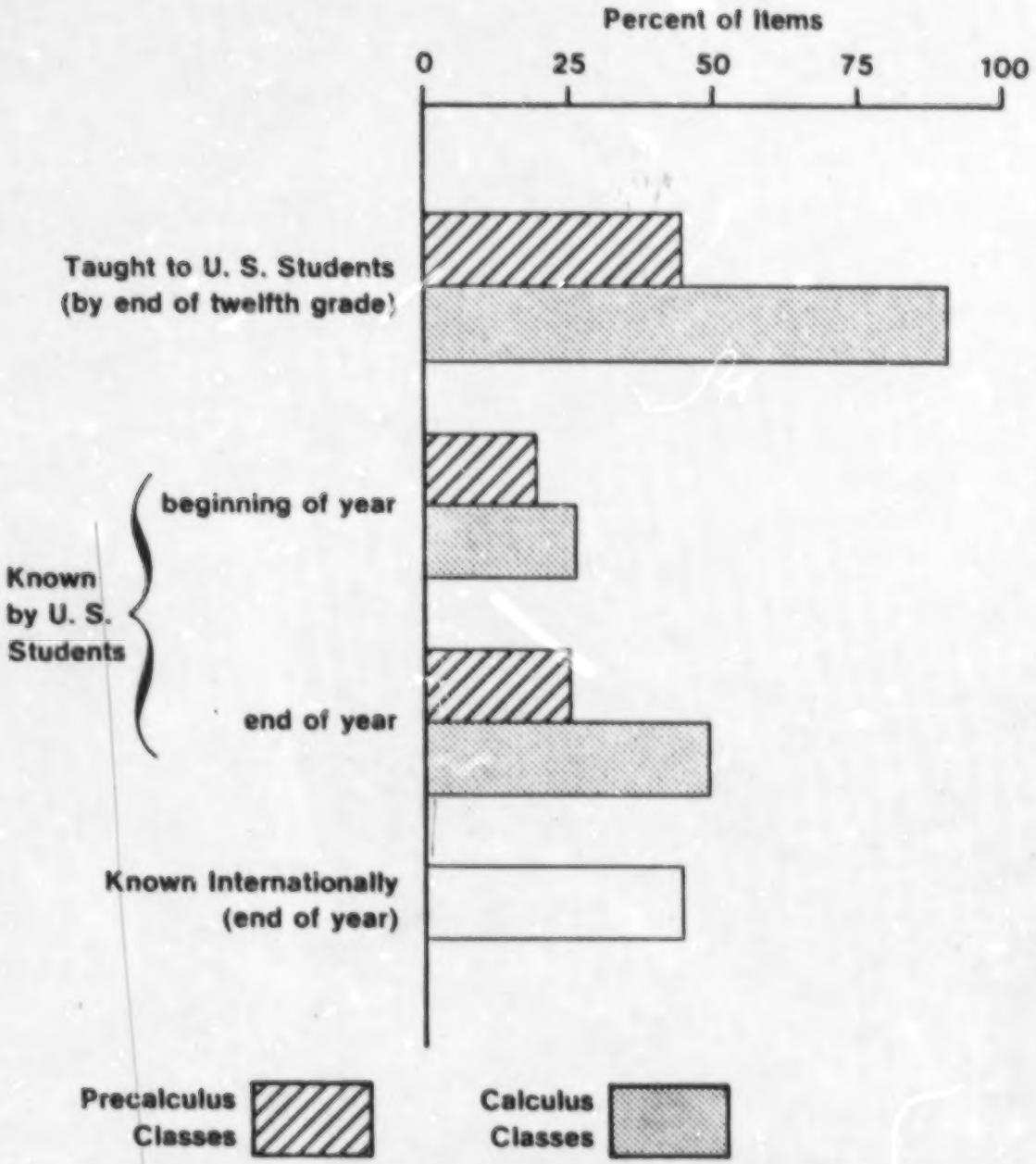


Figure 5 Average Percent of ELEMENTARY FUNCTIONS/CALCULUS Items on International Test Taught and Learned at End of Secondary School

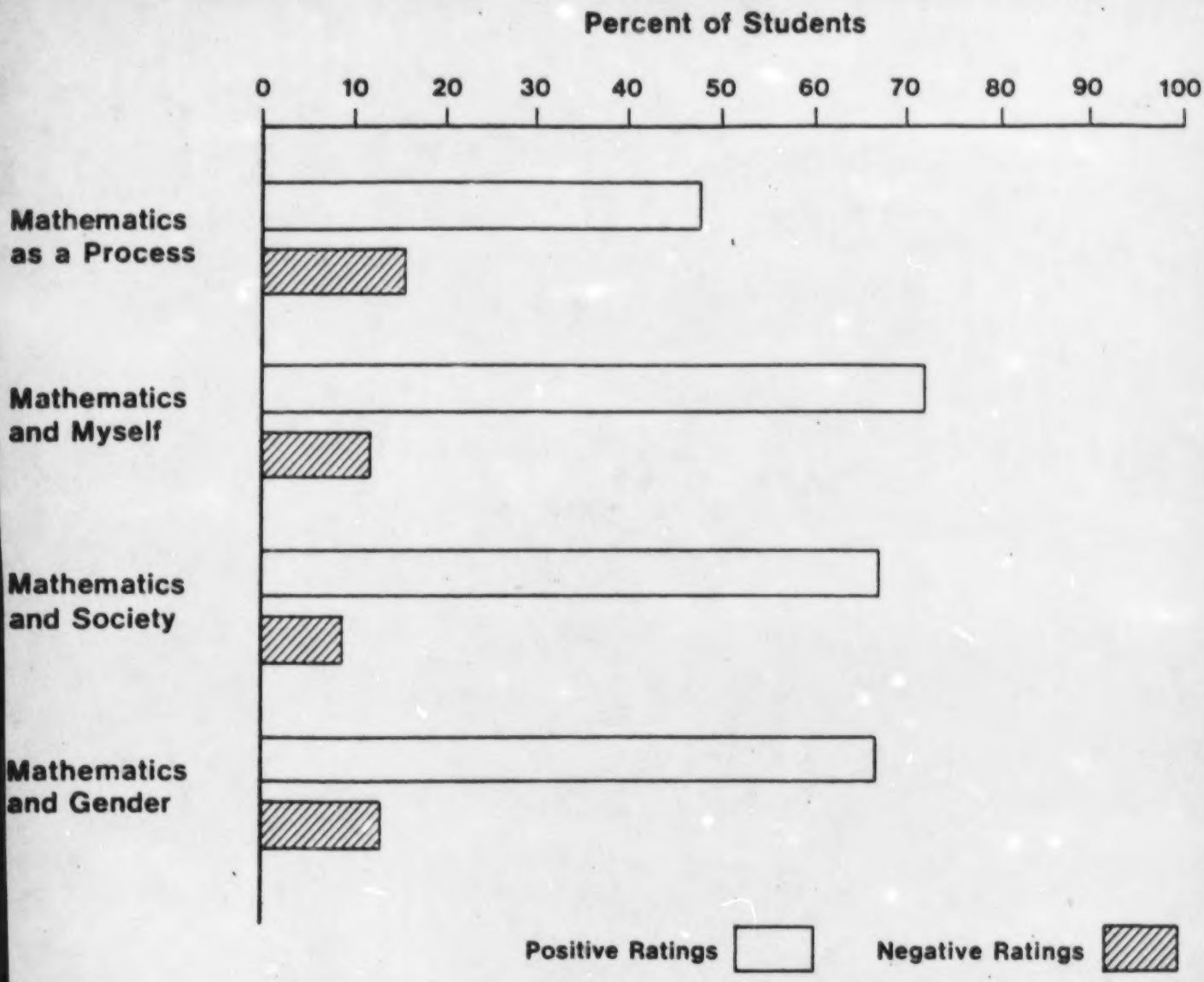
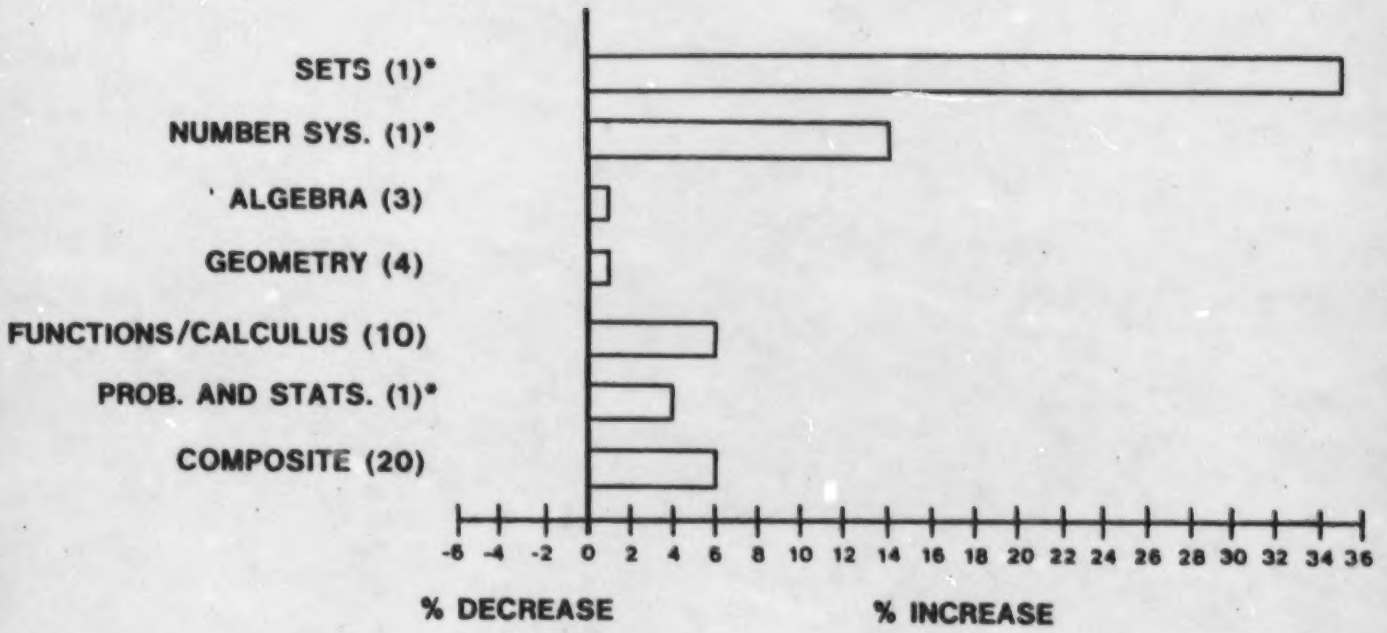


Figure 6 Percent Positive and Negative Ratings of Students for Attitude Scales in Twelfth Grade College-Preparatory Mathematics: U.S., 1981-82


SETS (1)*
NUMBER SYS. (1)*
ALGEBRA (3)
GEOMETRY (4)
FUNCTIONS/CALCULUS (10)
PROB. AND STATS. (1)*
COMPOSITE (20)
-6 -4 -2 0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36
% DECREASE
% INCREASE

*ONLY ONE ITEM IN THIS TOPIC AREA

FIGURE 7 POPULATION B - (TWELFTH GRADE IN USA)
CHANGES IN MEAN SCORES FROM 1964 TO 1982

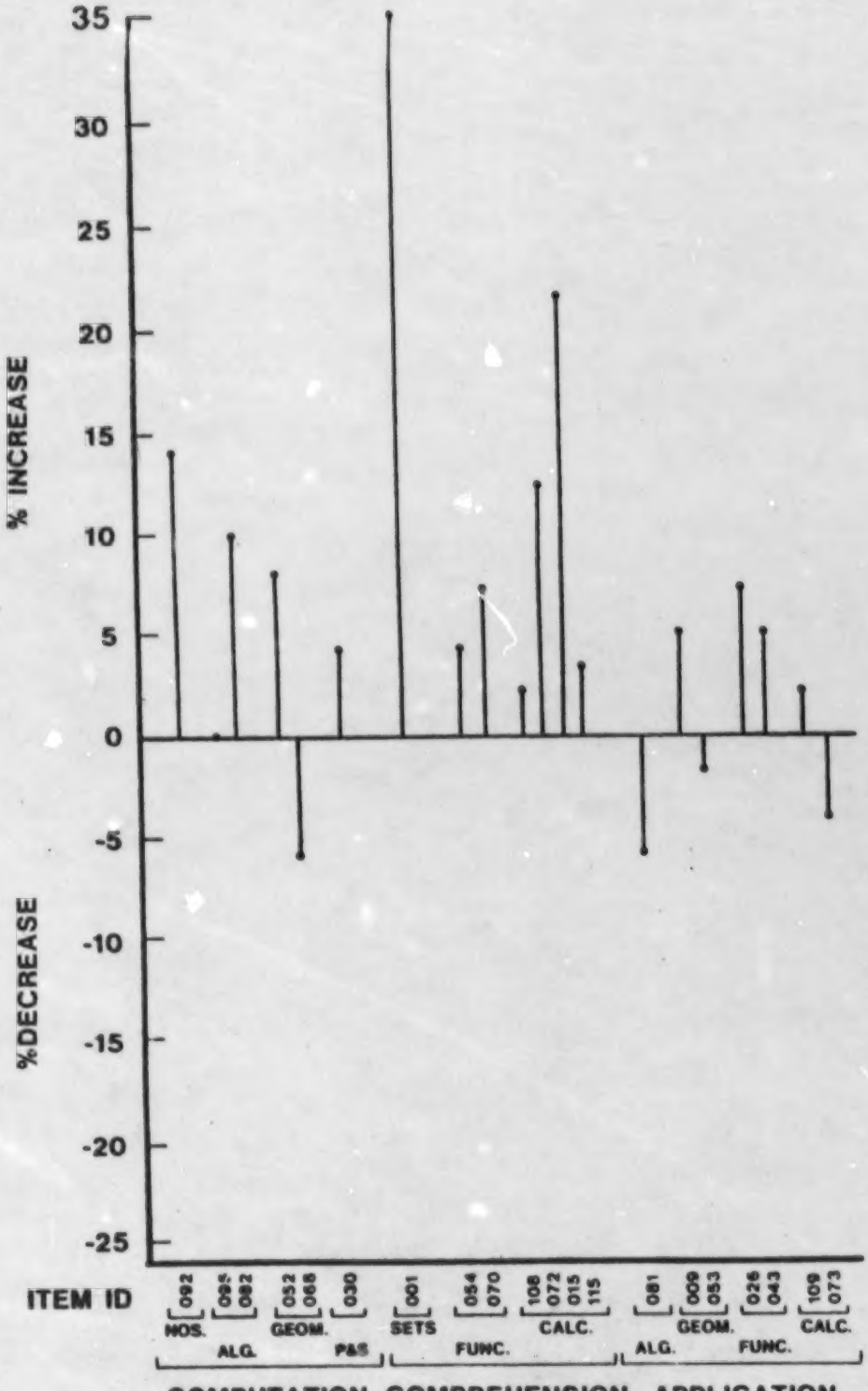


FIGURE 8 TWELFTH GRADE (USA)
STUDENT ACHIEVEMENT BY COGNITIVE LEVEL
CHANGE FROM 1964 TO 1982

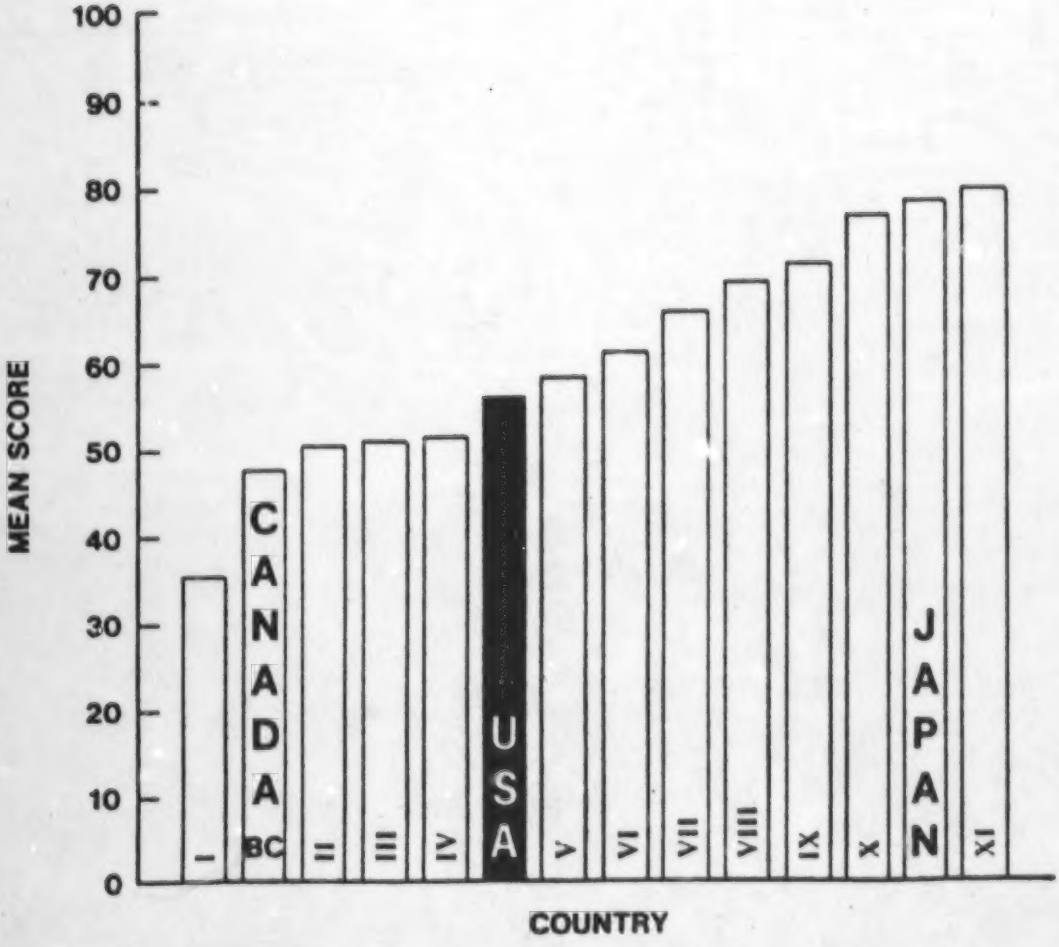


FIGURE 9 MEAN ACHIEVEMENT IN SETS AND RELATIONS FOR POPULATION B (TWELFTH GRADE IN U.S.) FOR 14 COUNTRIES

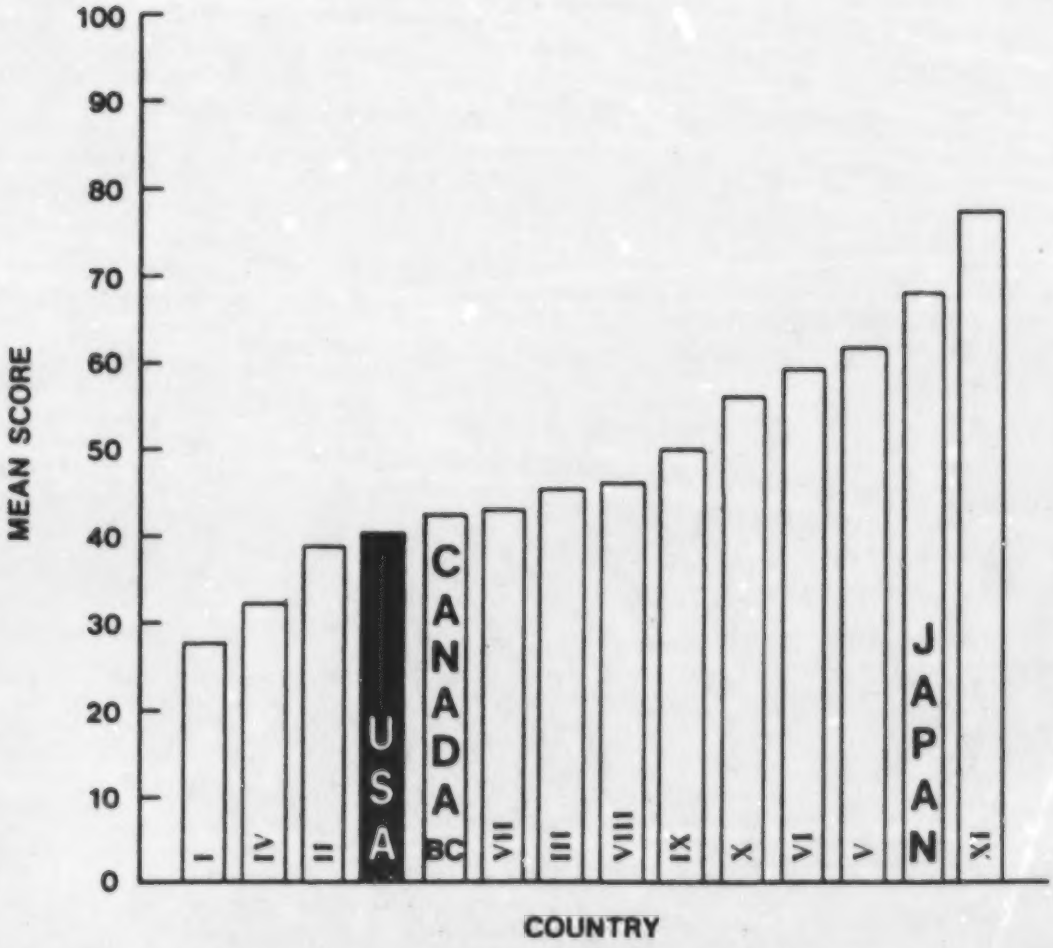


FIGURE 10 MEAN ACHIEVEMENT IN NUMBER SYSTEMS FOR POPULATION B (TWELFTH GRADE IN U.S.) FOR 14 COUNTRIES

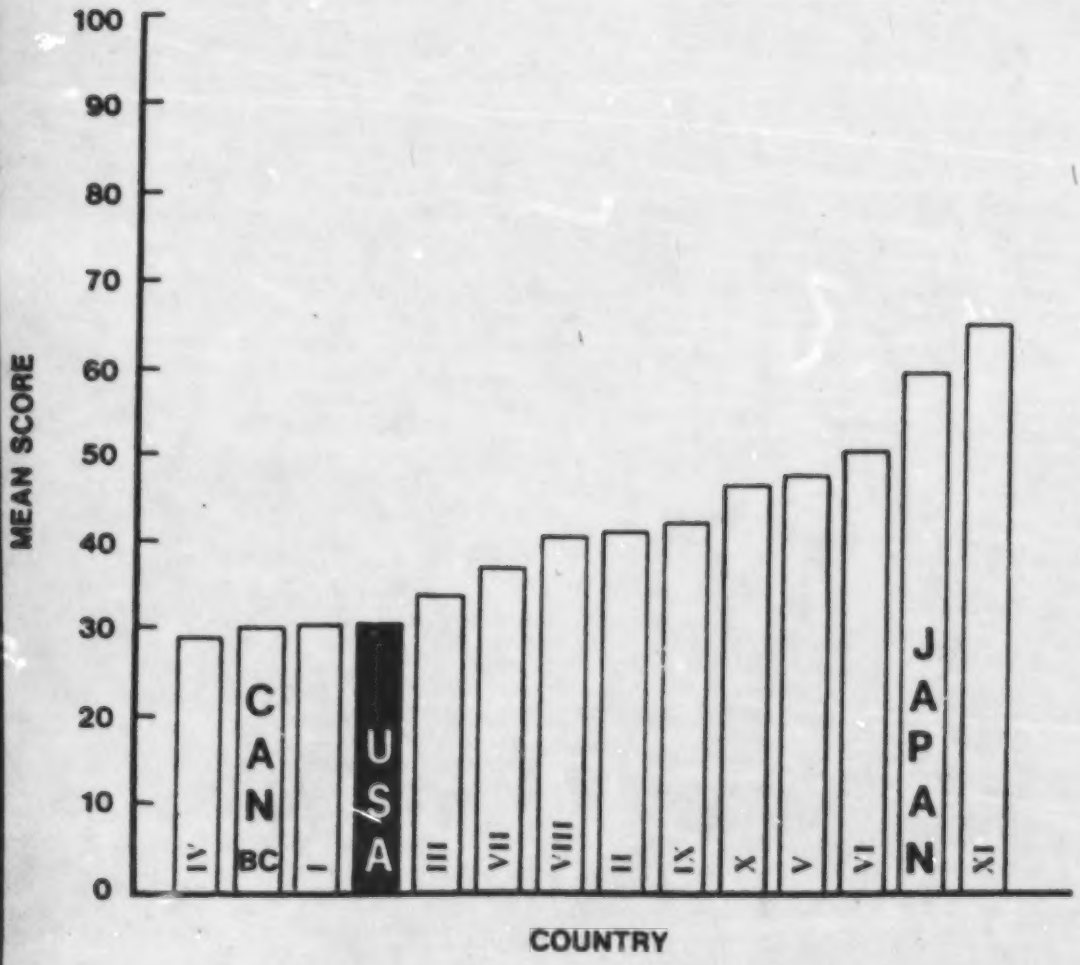


FIGURE 11 MEAN ACHIEVEMENT IN GEOMETRY FOR POPULATION B (TWELTH GRADE IN U.S.) FOR 14 COUNTRIES

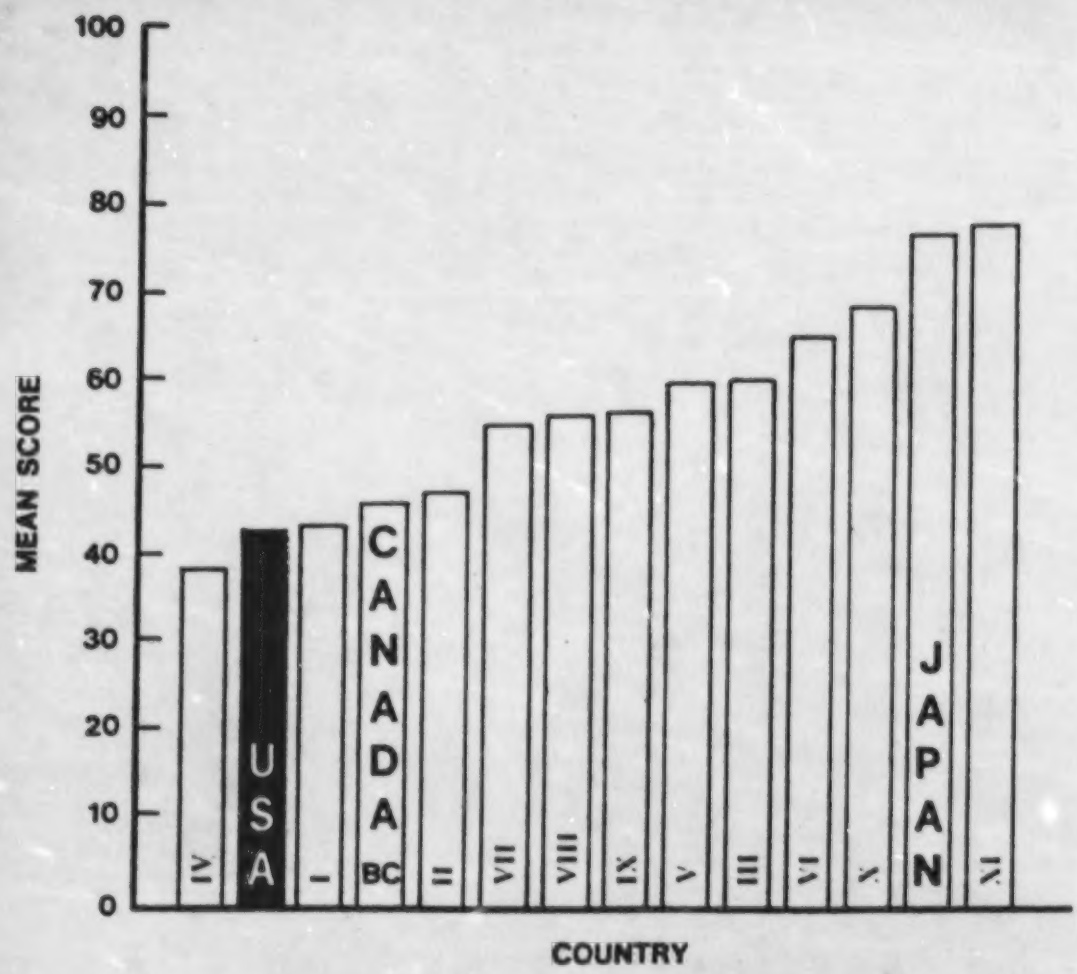


FIGURE 12 MEAN ACHIEVEMENT IN ALGEBRA FOR POPULATION B (TWELFTH GRADE IN U.S.) FOR 14 COUNTRIES

58

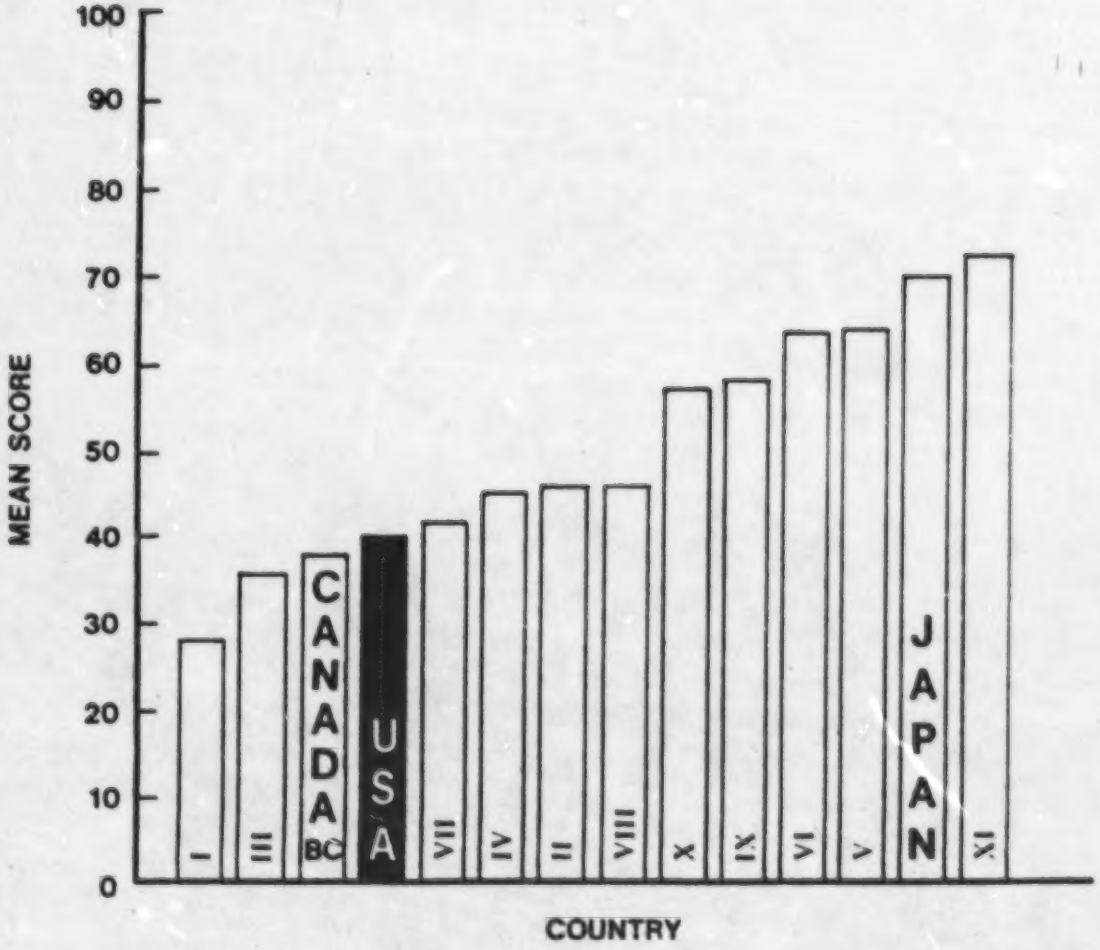


FIGURE 13 MEAN ACHIEVEMENT IN PROBABILITY AND STATISTICS FOR POPULATION B (TWELFTH GRADE IN U. S.) FOR 14 COUNTRIES

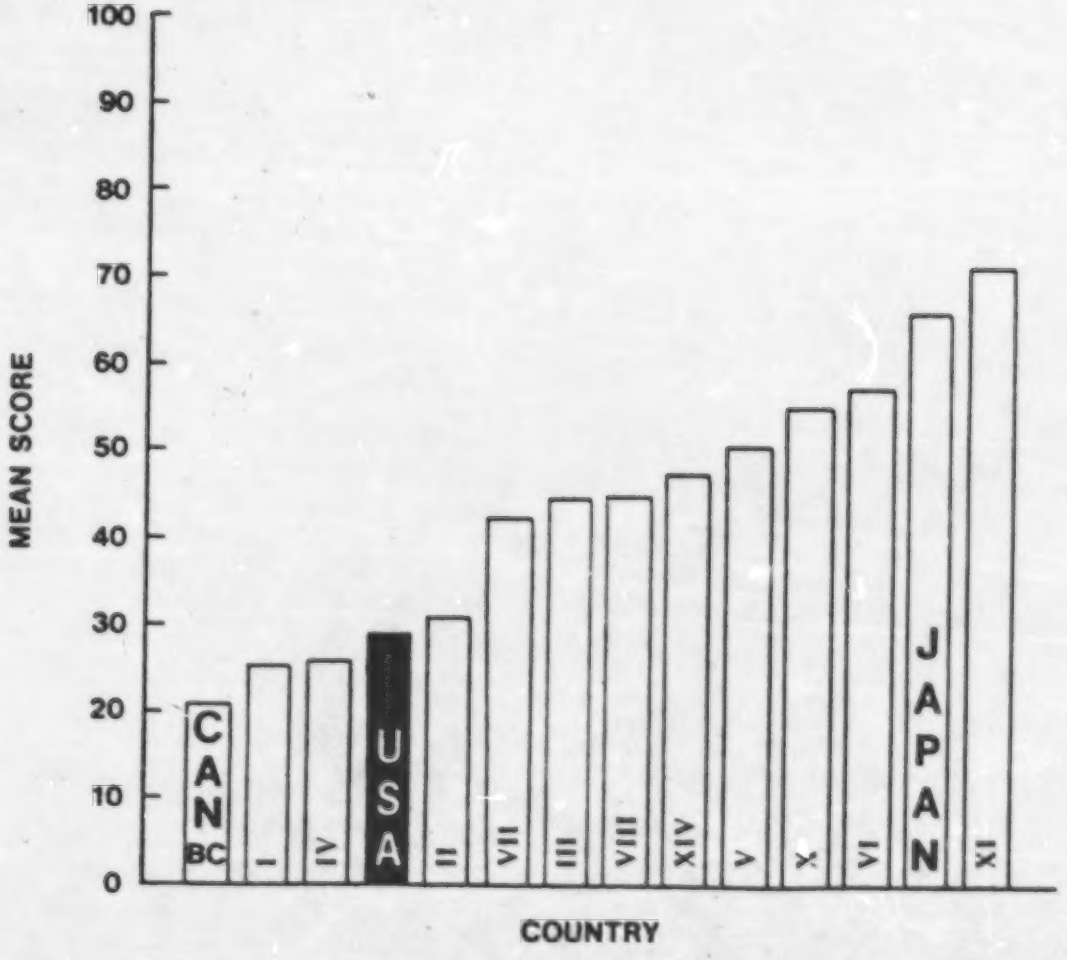


FIGURE 14 MEAN ACHIEVEMENT IN ELEMENTARY FUNCTIONS/CALCULUS FOR POPULATION B (TWELFTH GRADE IN U.S) FOR 14 COUNTRIES